Exam. Code : 211002
Subject Code : 4277

M.Sc. (Mathematics) 2nd Semester
MECHANICS—II
Paper—MATH-564

Time Allowed—Three Hours] [Maximum Marks—100

**Note :**—(1) This question paper consists of **EIGHT** questions (**FOUR** Sections).

(2) Attempt total **FIVE** questions.

(3) Candidates are required to attempt **FIVE** questions, selecting at least **ONE** question from each section i.e., the **fifth** question may be attempted from any section.

(4) All questions carry equal marks.

## SECTION—A

1. A rod mass M and length 2a, is rotating in a vertical plane with angular velocity $\omega$ about its center which is fixed. When the rod is horizontal its ascending end is struck by a ball of mass m which is falling with velocity u and when it is next horizontal the same end is struck by a similar ball falling with same velocity. The coefficient of resultant being unity, find the subsequent motion of the rod and balls.

2. (a) A uniform cylinder is placed with its axis horizontal on a plane, whose inclination to the horizontal is $\alpha$. Show that the least coefficient friction between it and the plane, so that it may roll and not slide, is $\frac{\tan\alpha}{3}$. Also if the cylinder be hollow and of small thickness the least value is $\frac{\tan\alpha}{2}$.

(b) Let AB and BC are two equal similar rods freely hanged at B and lie in a straight line on a smooth table. The end A is struck by a blow perpendicular to AB; show that the resulting velocity of A is 3½ times that of B.

## SECTION—B

3. (a) Derive Euler's dynamical equations for motion of a rigid body about a fixed point.

(b) Describe the motion of symmetric top.

4. (a) Prove that, Kinetic energy (T), Angular momentum (L) and magnitude of angular momentum $L^2$ of the force-free motion of a rigid body, are constants throughout the motion.

(b) If a rectangular parallelepiped with its edges 2a, 2a, 2b rotate about its centre of gravity under no forces. Prove that, its angular velocity about one principal axis is constant and about the axis it is periodic.

## SECTION—C

5. (a) Derive Lagrange's equation of motion for simple holonomic dynamical system.

   (b) For conservative system, show that the kinetic energy is a quadratic function of its generalized velocity.

6. Derive Lagrange's equation for simple pendulum and double pendulum.

## SECTION—D

7. (a) Distinctions between principle of least action and Hamilton's principle. A Particle of unit mass is projected so that its total energy is T in a field it is moving and the potential energy V(r) at a distance r from origin. Using the principle of least action, find the differential equation of the path.

   (b) Find the shortest distance between the points A(1, –1, 1) and B(2, 1, –1) lying on the surface $15x - 7y + z = 22$.

8. Use Rayleigh-Ritz method to minimize the integral :

$$I[y(x)] = \int_0^1 (-y^2 - 2xy)dx, \; y(0) = y(1) = 0 .$$